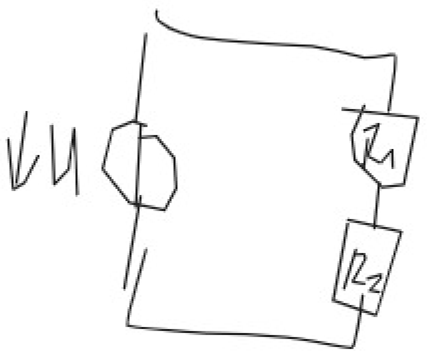
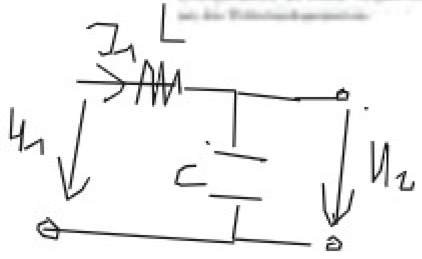
Spannungsteiler



$$\underline{U}_{R_2} = \frac{R_2}{R_1+R_2} \cdot \underline{U}$$

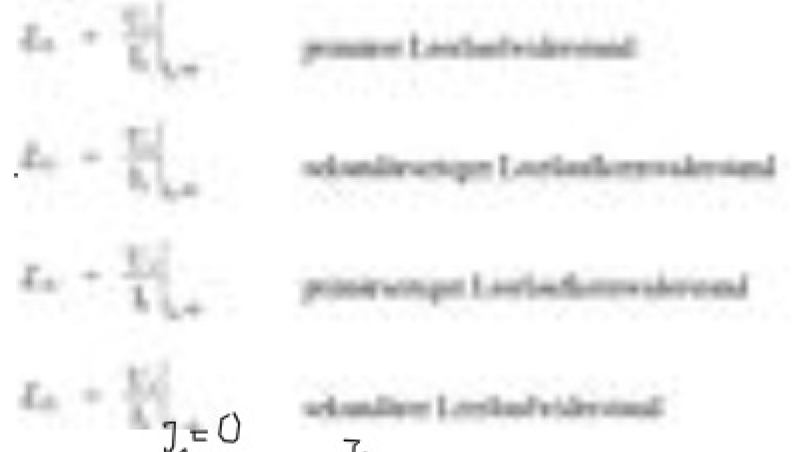
$\underline{U}$ $\underline{Z}$ $\underline{I}$

$\underline{U} =$



$$\underline{U}_2 = \frac{\underline{Z}_C}{\underline{Z}_L+\underline{Z}_C} \cdot \underline{U}_1$$

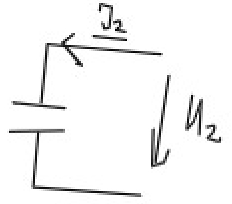
$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{\underline{U}_1}{\underline{Z}_C+\underline{Z}_L}$$



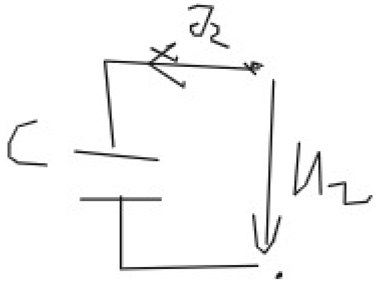
$I_1 = 0$

$$\underline{Z}_{11} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{U}_1}{\frac{\underline{U}_1}{\underline{Z}}} = \underline{Z} = \underline{Z}_L + \underline{Z}_C = j \cdot \left(2\pi f \cdot L - \frac{1}{2\pi f \cdot C}\right)$$

$$\underline{Z}_{12} = -j \cdot \frac{1}{2\pi f C}$$



$$\underline{Z}_{21} = \frac{\underline{U}_2}{\underline{I}_1} = \frac{\frac{\underline{Z}_C}{\cancel{\underline{Z}_L+\underline{Z}_C}} \cdot \cancel{\underline{U}_1}}{\frac{\cancel{\underline{U}_1}}{\cancel{\underline{Z}_C+\underline{Z}_L}}} = \underline{Z}_C$$



$$\underline{Z}_{22} = \underline{Z}_C$$

$\underline{U}$ $\underline{Z}$ $\underline{I}$

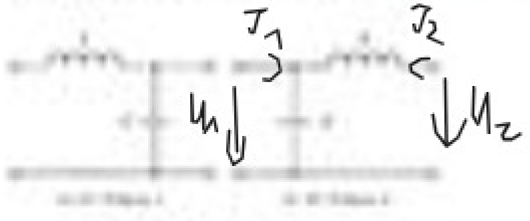
$$\frac{1}{j} = -j$$

$$\underline{U} = \underline{Z} \cdot \underline{I}$$

$$\underline{Z}_C = \frac{1}{2\pi \cdot f \cdot C \cdot j}$$

$$\underline{Z}_L = 2\pi \cdot f \cdot L \cdot j$$

$$\underline{Z} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} \underline{Z}_L+\underline{Z}_C & \underline{Z}_C \\ \underline{Z}_C & \underline{Z}_C \end{pmatrix}$$

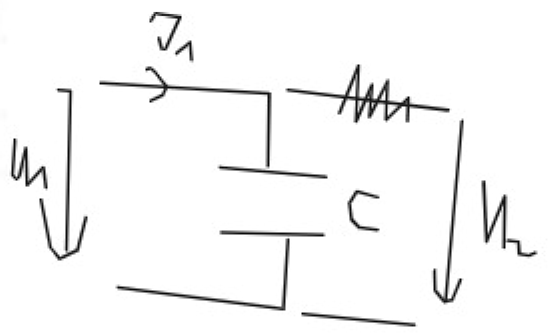


$$\Omega = \frac{\omega}{\omega_0} \Rightarrow$$

$$\Rightarrow \omega = \omega_0 \cdot \Omega = \frac{\Omega}{\sqrt{L \cdot C}}$$

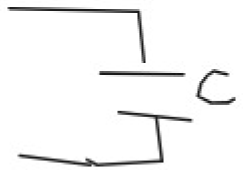
$$Z_L = \omega \cdot L \cdot j = \frac{\Omega}{\sqrt{L \cdot C}} \cdot L \cdot j = \Omega \cdot \sqrt{\frac{L^2}{L \cdot C}} \cdot j = \Omega \cdot \sqrt{\frac{L}{C}} \cdot j = \Omega \cdot Z_0 \cdot j$$

$$Z_C = -\frac{1}{\omega \cdot C} \cdot j = -\frac{1}{\frac{\Omega}{\sqrt{L \cdot C}} \cdot C} \cdot j = -\frac{1}{\Omega \cdot \sqrt{\frac{C^2}{L \cdot C}}} \cdot j = -\frac{1}{\Omega \cdot \sqrt{\frac{C}{L}}} \cdot j = -\frac{1}{\Omega \cdot \frac{1}{Z_0}} \cdot j = -\frac{Z_0}{\Omega} \cdot j$$

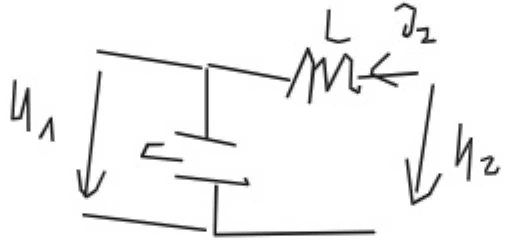


$$\underline{Z_{21}} = \underline{Z_C}$$

$$\underline{Z_{22}} = \underline{Z_C} + \underline{Z_L}$$



$$Z_{11} = Z_C$$



$$Z_{12} = \frac{U_1}{J_2} = \frac{\frac{Z_C}{Z_C + Z_L} \cdot U_2}{\frac{U_2}{Z_C + Z_L}} = Z_C$$

$$U_1 = \frac{Z_C}{Z_C + Z_L} \cdot U_2$$

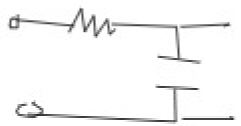
$$J_2 = \frac{U_2}{Z_C + Z_L}$$

$$Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} Z_C & Z_C \\ Z_C & Z_C + Z_L \end{pmatrix}$$

$$= -\frac{1}{\Omega\cdot\sqrt{\frac{C}{L}}\cdot j} = -\frac{1}{\Omega\cdot\frac{1}{Z_0}\cdot j} = -\frac{Z_0}{\Omega\cdot j}$$

In Matrixform umgeschrieben lautet die Darstellung in Kettenparametern:

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{Z_{11}}{Z_{21}} & -\frac{\det[Z]}{Z_{21}} \\ \frac{1}{Z_{21}} & -\frac{Z_{22}}{Z_{21}} \end{pmatrix}}_{=A} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

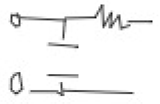


$$Z = \begin{pmatrix} Z_L+Z_C & Z_C \\ Z_C & Z_C \end{pmatrix}$$

$$A_1 = \begin{pmatrix} \frac{Z_L+Z_C}{Z_C} & -\frac{Z_L\cdot Z_C}{Z_C} \\ \frac{1}{Z_C} & -\frac{Z_C}{Z_C} \end{pmatrix} = \begin{pmatrix} \frac{Z_L}{Z_C}+1 & -Z_L \\ \frac{1}{Z_C} & -1 \end{pmatrix}$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad-bc$$

$$\det Z = (Z_L+Z_C)\cdot Z_C - Z_C^2 = Z_L\cdot Z_C$$



$$Z = \begin{pmatrix} Z_C & Z_C \\ Z_C & Z_L+Z_C \end{pmatrix}$$

$$A_2 = \begin{pmatrix} \frac{Z_C}{Z_C} & -\frac{Z_L\cdot Z_C}{Z_C} \\ \frac{1}{Z_C} & -\frac{Z_L+Z_C}{Z_C} \end{pmatrix} = \begin{pmatrix} 1 & -Z_L \\ \frac{1}{Z_C} & -(1+\frac{Z_L}{Z_C}) \end{pmatrix}$$

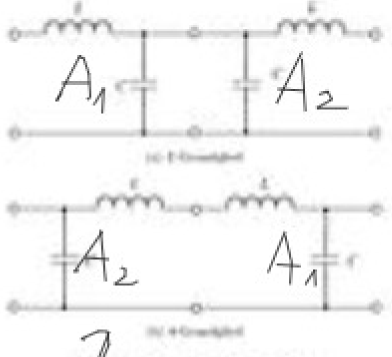


Abbildung 2: Tiefpass-Grundglieder

$$A = A_1\cdot A_2 = \begin{pmatrix} \frac{Z_L}{Z_C}+1 & -Z_L \\ \frac{1}{Z_C} & -1 \end{pmatrix}\begin{pmatrix} 1 & -Z_L \\ \frac{1}{Z_C} & -(1+\frac{Z_L}{Z_C}) \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{Z_L}{Z_C}+1-\frac{Z_L}{Z_C} & (\frac{Z_L}{Z_C}+1)\cdot(-Z_L)+Z_L\cdot(1+\frac{Z_L}{Z_C}) \\ \frac{1}{Z_C}-\frac{1}{Z_C} & -\frac{Z_L}{Z_C}+1+\frac{Z_L}{Z_C} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = A_2\cdot A_1 = \begin{pmatrix} 1 & -Z_L \\ \frac{1}{Z_C} & -(1+\frac{Z_L}{Z_C}) \end{pmatrix}\cdot\begin{pmatrix} \frac{Z_L}{Z_C}+1 & -Z_L \\ \frac{1}{Z_C} & -1 \end{pmatrix} = \begin{pmatrix} \frac{Z_L}{Z_C}+1-\frac{Z_L}{Z_C} & -Z_L+Z_L \\ (\frac{Z_L}{Z_C}+1)\cdot\frac{1}{Z_C}-(1+\frac{Z_L}{Z_C})\cdot\frac{1}{Z_C} & \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

6.9.1 Serienschaltung (Kettenschaltung) von Teilvierpolen

In Abbildung 6-6 ist die Kettenschaltung von n Teilvierpolen zu sehen. Da wir die Kettendarstellung der Vierpole verwenden, wählt man eine unsymmetrische Bepfeilung.

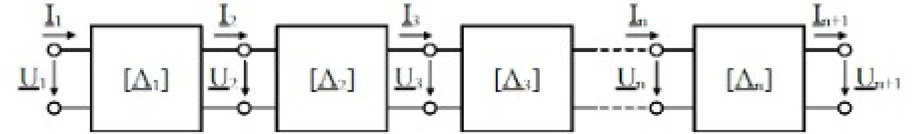


Abbildung 6-6: Kettenschaltung von n Teilvierpolen

Die Größen am Ausgang des Vierpols 2 berechnen sich aus dem Produkt der Kettenmatrizen:

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = [A_1]\begin{pmatrix} U_2 \\ I_2 \end{pmatrix} \qquad \begin{pmatrix} U_2 \\ I_2 \end{pmatrix} = [A_2]\begin{pmatrix} U_3 \\ I_3 \end{pmatrix} \tag{6-41}$$

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = [A_1][A_2]\begin{pmatrix} U_3 \\ I_3 \end{pmatrix}$$

c) Der Eingangswiderstand $\underline{Z}_e$ läßt sich als Funktion der Kettenparameter und des Abschlusswiderstandes $\underline{Z}_2$ gemäß

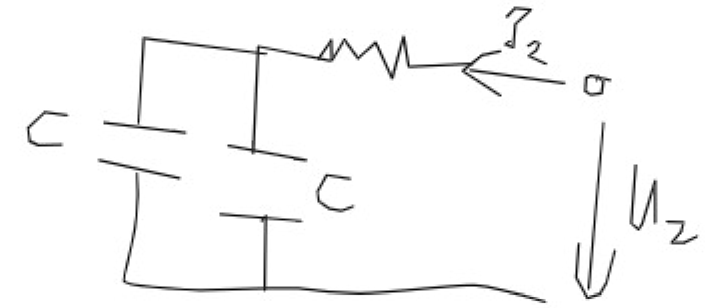
$$\underline{Z}_e = \frac{\underline{U}_1}{\underline{I}_1} = \frac{A_{11}\underline{Z}_2 + A_{12}}{A_{21}\underline{Z}_2 + A_{22}}$$

darstellen.
Es gibt nun einen Abschlusswiderstand $\underline{Z}_2$, bei dem der Eingangswiderstand $\underline{Z}_e = \underline{Z}_2$ wird. Berechnen Sie diesen Widerstand für die Schaltungen aus (Abbildungen 2(a) und 2(b)) als Funktion des Kennwiderstandes und der normierten Frequenz.
Beschreiben Sie in Worten, was an der Stelle $\Omega = 1$ passiert.

$$Z_e \overset{!}{=} Z_2$$

$$\frac{A_{11} Z_2 + A_{12}}{A_{21} Z_2 + A_{22}} = Z_2$$

$$\frac{1 \cdot Z_2 + 0}{0 \cdot Z_2 + 1} = Z_2$$



$$\underline{Z_2} = \underline{Z_L} + \frac{1}{2}\underline{Z_C}$$

$$= \Omega \cdot Z_0 \cdot j + \frac{1}{2} \cdot -\frac{Z_0}{\Omega} \cdot j$$

$$= Z_0 \cdot j \cdot \left(\Omega - \frac{1}{2\Omega}\right)$$

$$\frac{|Z_2|}{Z_0} = \frac{\cancel{Z_0} \cdot \left|\Omega - \frac{1}{2\Omega}\right|}{\cancel{Z_0}} = \underline{\underline{\left|\Omega - \frac{1}{2\Omega}\right|}}$$

$$\Omega - \frac{1}{2\Omega} = 0 \quad |$$

$$\Omega = \frac{1}{2\Omega} \quad | \cdot 2\Omega$$

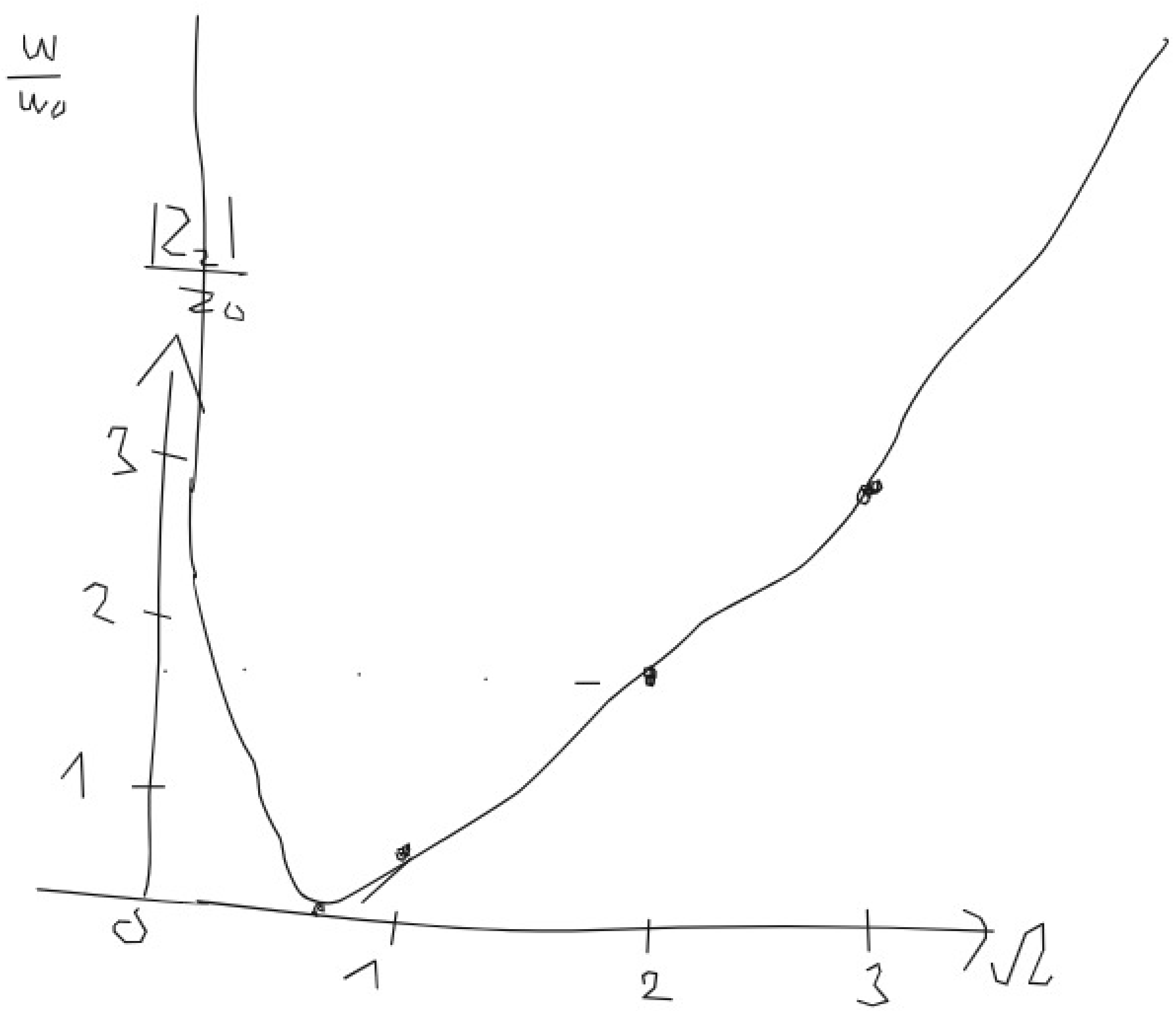
$$2\Omega^2 = 1 \quad | :2$$

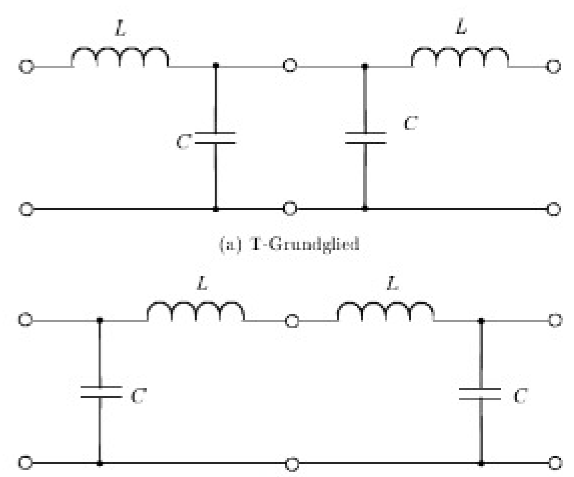
$$\Omega = \sqrt{\tfrac{1}{2}} \approx 0{,}7$$

d) Stellen Sie den normierten Betrag $|\underline{Z}_2|/\underline{Z}_0$ über der normierten Frequenz $(0 < \Omega < 3)$ auf einer linearen Skala dar.

Beschreiben Sie in Worten den Unterschied zwischen dem T- und dem π-Glied.

$$\Omega = \frac{\omega}{\omega_0}$$



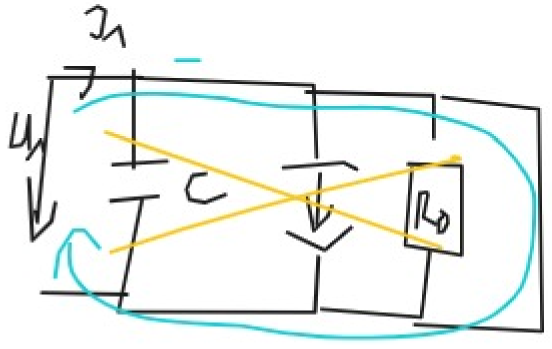


(a) T-Grundglied

(b) π-Grundglied

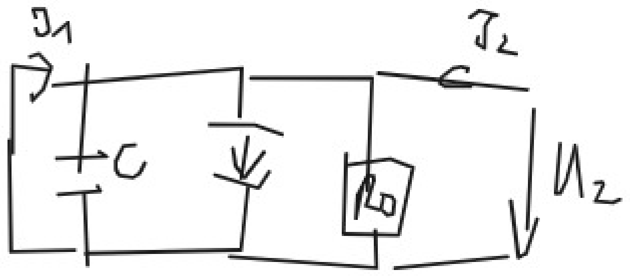
Abbildung 2: Tiefpass-Grundglieder.

$$G = \frac{I}{U}$$



$I_1 = +\infty$

$Y_{11} = +\infty$



$I_1 = -\infty$ $\underline{Y_{12}} = -\infty$